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Hydrostatic Levitation of Particles in Electric and Magnetic Fields

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Abstract

Nonhomogeneous magnetic and electric fields generate levitational forces in magnetic and dielectric liquids whereby particles are separated according to their specific gravity and magnetic and electric properties, respectively. The corresponding systems are called magnetohydrostatic separation and wet dielectric separation. These forces depend on the field intensity and gradient, and on properties of the liquid and the particles to be levitated. This paper deals with the potential application of electric fields as compared with that of the magnetic field. It is shown that unless special measures are taken, the electric field is inferior to such an extent that its application is not promising. In this context the influences of particle size and shape in both methods are discussed.

INTRODUCTION

The magnetohydrostatic separation (M.H.S.) (*1*, *2*) and wet dielectric separation (W.D.S.) (*3–5*) methods make use of the same principles inasmuch as the levitational forces are generated within a magnetic or a dielectric liquid in an inhomogeneous field. These forces, combined with gravity, can lead to the separation of particles according to their specific gravities and their magnetic, or dielectric, properties. The analogy between the magnetic (free of currents) and electric (free of charges) potential fields suggests that the electric field, being simpler to obtain, can be used in a manner similar to the magnetic field in M.H.S.

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It was found that because of the physical properties of dielectrics, the levitational forces obtainable are generally too small to be of practical value. This is supported by the fact that the energy density obtainable in magnetic fields is substantially higher than that in electric fields. Furthermore, the choice of the liquid to be used in W.D.S. is a difficult task. Two conflicting parameters determine which dielectric liquid is to be given preference: a high dielectric constant will provide a high contrast force between the liquid and the particles, but it reduces the other parameter, viz., the available levitation force.

This paper discusses the basic difference between magnetic and electric fields as regards their use, actual or potential, in gravity (levitational) separation.

LEVITATION FORCES IN THE M.H.S. AND W.D.S. METHODS

The magnetic and the electric forces per unit volume of a spherical particle immersed in a magnetic or a dielectric liquid in a nonhomogeneous field are given by (*1–3*):

$$\mathbf{f}_m = 3\mu_l\gamma_m\nabla(H^2)/(8\pi) \tag{1}$$

$$\mathbf{f}_e = 3\varepsilon_l\gamma_e\nabla(E^2)/(8\pi) \tag{2}$$

where

$$\gamma_m = (\mu_l - \mu_p)/(2\mu_l + \mu_p)$$

and

$$\gamma_e = (\varepsilon_l - \varepsilon_p)/(2\varepsilon_l + \varepsilon_p)$$

The other terms are defined in the Symbols section.

As the magnetic field is free of currents and the electric field free of charges, the divergences of both fields are zero and, the field being defined as the negative gradient of the potential, Laplace's equation is obtained for the potential.

One type of the potential solution for both fields is given by (*2*)

$$\phi = Ar^k \sin(K\theta) \tag{3}$$

This potential is produced by a curved configuration of the pole pieces and the electrodes for the magnetic and the electric fields, respectively, as in Fig. 1. Substituting in Eqs. (1) and (2) the respective fields obtained

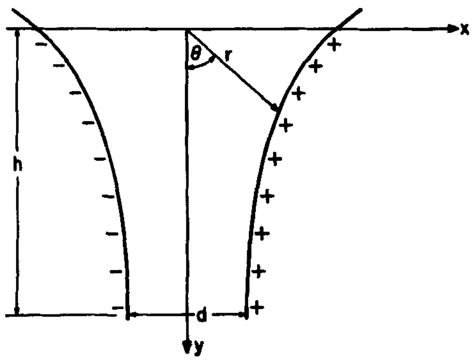

FIG. 1. Curved configuration of electrodes.

from Eq. (3), we obtain

$$\mathbf{f}_m = \frac{3\mu_l\gamma_m}{4\pi}A_m{}^2K^2(K - 1)r^{2k-3}\mathbf{1}_r \tag{4}$$

and

$$\mathbf{f}_e = \frac{3\varepsilon_l\gamma_e}{4\pi}A_e{}^2K^2(k - 1)r^{2k-3}\mathbf{1}_r \tag{5}$$

where $\mathbf{1}_r$ is a unit vector colinear with $\mathbf{r}$, and A^2 is a constant determined by the field intensity in the air gap d (Fig. 1) and the configuration shape, and given by

$$A_m = H_0/(Kh^{k-1}\mu_l) \tag{6}$$

$$A_e = E_0/(Kh^{k-1}\varepsilon_l) \tag{7}$$

When $\theta = 0$ (Fig. 1), then $\mathbf{1}_r$ is colinear with y, and the comparison between the levitational forces can be made with the aid of Eqs. (4) and (5), in which they differ by a factor C. C is given by

$$C_m = 3\mu_l\gamma_m A_m{}^2/(4\pi) \tag{8}$$

$$C_e = 3\varepsilon_l\gamma_e A_e{}^2/(4\pi) \tag{9}$$

These differences in levitational forces are summarized in Table 1 for the following data concerning both fields:

(a) $h = 5$ cm, length of pole pieces or electrodes

TABLE 1

Magnetic and Electric Levitational Forces as Function of Various Parameters.

K	Pole piece or electrode shape	γ_m	H_0 (kG)	A_m^2	γ_e	E_0 (statvolt/cm)	A_e^2	$\frac{C_m}{C_e}$	ρ_m'[a] (g/cm³)	ρ_e' (g/cm³)
2	Hyperbolic (Fig. 2)	3.35×10^{-4}	20.0	4×10^6 $\frac{\text{gauss}^2}{\text{cm}^2}$	0.227	133.3	2.2 $\frac{\text{statvolt}^2}{\text{cm}^4}$	298.5	6.4	2.1×10^{-2}
1.5	Isodynamic (Fig. 3)	3.35×10^{-4}	20.0	3.55×10^7 $\frac{\text{gauss}^2}{\text{cm}}$	0.227	133.3	19.49 $\frac{\text{statvolt}^2}{\text{cm}^3}$	298.9	3.18	10^{-2}
2.5	(−) Fig. 4	3.35×10^{-4}	20.0	5.12×10^5 $\frac{\text{gauss}^2}{\text{cm}^3}$	0.227	133.3	0.28 $\frac{\text{statvolt}^2}{\text{cm}^5}$	301.4	9.59	3.18×10^{-2}

[a]In the three given pole piece, and electrode, configurations ρ' provides the greatest contribution to the apparent density of the liquid, ρ_e' being two orders of magnitude smaller than ρ_m'.

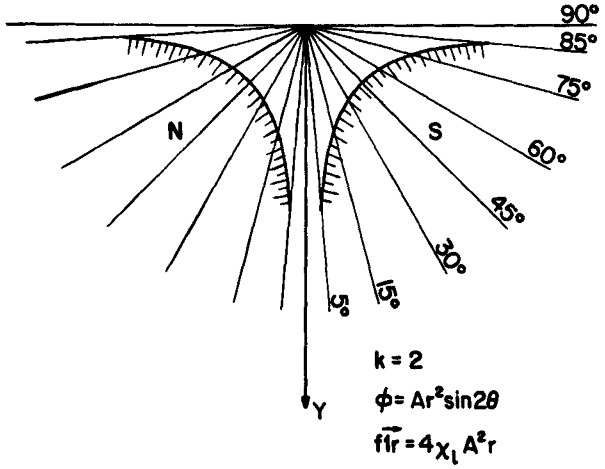

FIG. 2. Hyperbolic pole configuration.

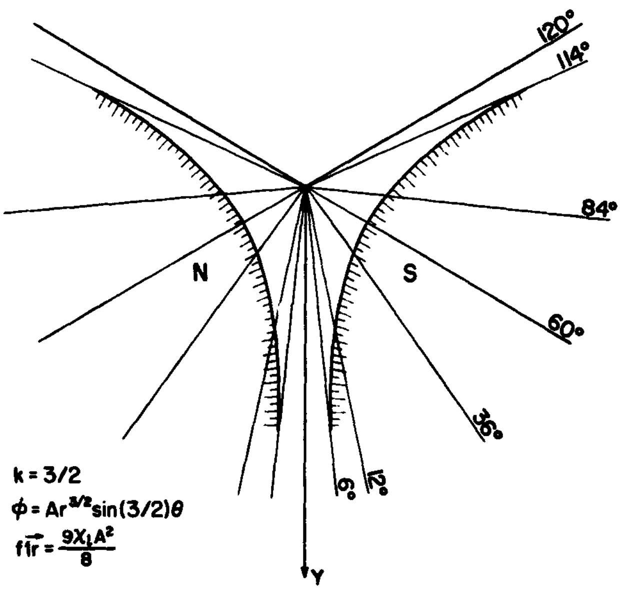

FIG. 3. Isodynamic pole configuration.

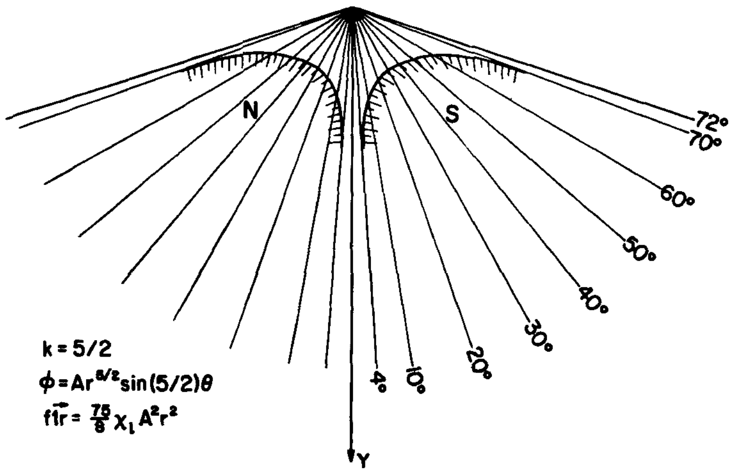

FIG. 4. Curved configuration for $k = 5/2$.

(b) $d = 0.5$ cm, air gap
(c) $\chi_l = 8 \times 10^{-5}$ emu/cm^3, for a saturated solution of $MnCl_2 \cdot 4H_2O$, where $\mu_l = 1 + 4\pi\chi_l$
(d) $\chi_p = 0$, for diamagnetic or weakly paramagnetic particles
(e) $\varepsilon_l = 9$, for dichloromethane (CH_2Cl_2)
(f) $\varepsilon_p = 4$, for glass beads

EFFECT OF PARTICLE SIZE IN THE M.H.S. SYSTEM

As was said before, the levitational force produced in the M.H.S. method depends mainly on the intensity of the magnetic field and its gradient, such that particle size becomes one of the important parameters in a high-gradient system. In order to observe the influence of particle size on the levitation force, we shall choose a sphere immersed in a paramagnetic liquid and held between curved poles of the configuration shown in Fig. 5.

According to Eq. (4), the magnetic force acting on this spherical particle, in the y direction, is the integral of all the forces acting over the entire particle volume, given by

$$(F_m)_y = (\chi_l - \chi_p)A_m{}^2K^2(K-1)\int_v r^{2k-3}\cos\alpha\, dv \qquad (10)$$

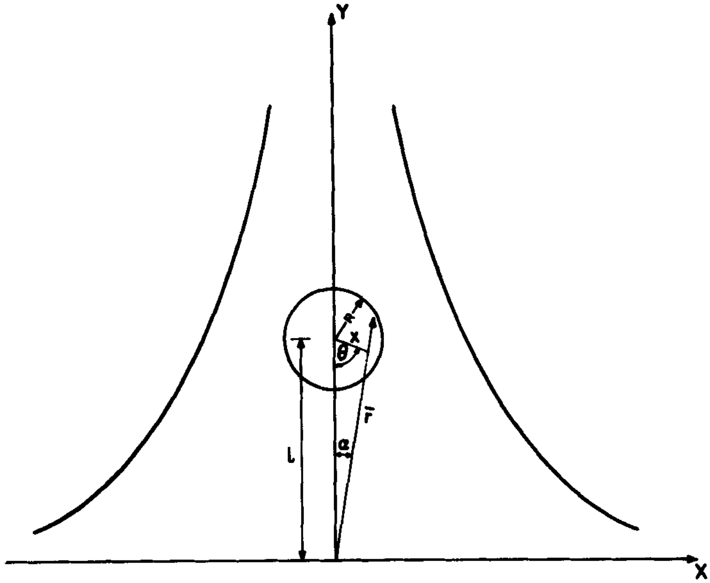

FIG. 5. Spherical particle in curved pole configuration.

Equation (10), with the geometrical relations in Fig. 5, yields

$$(F_m)_y = (\chi_l - \chi_p)A_m{}^2K^2(K-1)\int_0^\pi (l^2 + x^2 - 2xl\cos\theta)^{k-2} \times (l - x\cos\theta)\sin\theta\, d\theta \int_0^R x^2\, dx \int_0^{2\pi} d\phi \tag{11}$$

Solving Eq. (11) (see Appendix A), we find (2)

$$(F_m)_y = \frac{\pi(\chi_l - \chi_p)A_m{}^2k}{2}\left[\frac{(l-R)^{2(k+1)} - (l+R)^{2(k+1)}}{2l^2(k+1)} + \frac{(l+R)^{2k+1} - (l-R)^{2k+1}}{l} + (l-R)^{2k} - (l+R)^{2k}\right] \tag{12}$$

For the hyperbolic pole pieces $K = 2$, as shown in Fig. 2, and Eq. (12) reduces to

$$(F_m)_y = \frac{16\pi R^3}{3}(\chi_l - \chi_p)A_m{}^2 l = 4V(\chi_l - \chi_p)A_m{}^2 l \tag{13}$$

regardless of particle size.

For the isodynamic pole pieces $K = 3/2$, as shown in Fig. 3, and Eq. (12) reduces to

$$(F_m)_y = \frac{9}{8}\frac{4\pi R^3}{3}(\chi_l - \chi_p)A_m{}^2(1 - R^2/5l^2) \tag{14}$$

It is seen from Eq. (14) that the behavior is truly isodynamic only when $R/l \to 0$. When this value is larger, the magnetic force acts as a screen. In practice this value is negligible because $R = 0.1$ to 1.0 mm, and $l = 5$ to 10 cm, so that $R/l = 1/50$.

EFFECT OF PARTICLE SHAPE IN THE W.D.S. SYSTEM

The electric force per unit volume of a spherical particle immersed in a dielectric liquid in the system shown in Fig. 6 is given by Eq. (5). For a hyperbolic electrode configuration ($K = 2$), Eq. (3)—which in this system gives the electric potential—becomes

$$\phi = A_e r'^2 \sin 2\theta' \tag{15}$$

$E_0{}^2$ within the dielectric liquid is derived from Eq. (15) as

$$E_0{}^2 = 4A_e{}^2 r'^2 \varepsilon_l{}^2 \tag{16}$$

If the particle size is very small compared with the gap between the electrodes, a homogeneous field may be assumed to exist near the particle. The solution for the electric field around the particle is then given by (*6*)

$$E_r = E_0(1 - 2\gamma_e(a/r)^3)\cos\theta \tag{17}$$

$$E_\theta = -E_0(1 + \gamma_e(a/r)^3)\sin\theta \tag{18}$$

and E^2 is given by

$$E^2 = E_0{}^2[(1 + 2\gamma_e(a/r)^3(1 - 3\cos^2\theta)) + 0(\gamma_e{}^2)] \tag{19}$$

According to the geometrical relations in Fig. 6 between r', θ', r, and θ, Eq. (19) reduces to

$$E^2 = 4A_e{}^2(r^2 + h^2 - 2rh\sin\theta)(1 + 2\gamma_e(a/r)^3(1 - 3\cos^2\theta)) \tag{20}$$

The net electric force acting on the particle immersed in the dielectric liquid is given by the surface integral:

$$(F_e)_y = \oint (p\cdot n)\, ds \tag{21}$$

where n is a unit vector perpendicular to the particle surface.

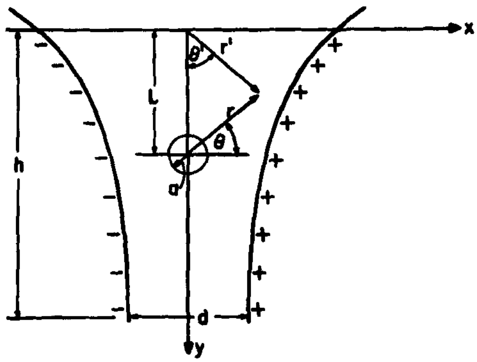

FIG. 6. Spherical particle in hyperbolic electrodes.

Substituting Eq. (20) into Eq. (21) yields

$$(F_e)_y = \frac{3\varepsilon_l \gamma_e A_e^2 a^2}{2\pi} \int_0^{2\pi} d\phi \int_0^{\pi} (a^2 + h^2 - 2al\cos\theta) \times (1 + 2\gamma_e - 6\gamma_e \sin^2\theta)\sin\theta\cos\theta\, d\theta \qquad (22)$$

and the solution of the integral (see Appendix B) yields

$$(F_e)_y = \left(\frac{3}{2\pi}\right) V\varepsilon_l \gamma_e A_e^2 l(2.8 - 4\gamma_e) \qquad (23)$$

Substituting Eq. (7) into Eq. (23) yields

$$(F_e)_y = \frac{\beta\gamma_e}{\varepsilon_l}(2.8 - 4\gamma_e) \qquad (24)$$

where $\beta = 3VE_0^2 l/8\pi h^2$ = constant.

For a spherical particle

$$\gamma_e = \frac{\varepsilon_l - \varepsilon_p}{2\varepsilon_l + \varepsilon_p}$$

and for a cylindrical particle

$$\gamma_e = \frac{\varepsilon_l - \varepsilon_p}{\varepsilon_l + \varepsilon_p}$$

from which it follows that particle shape has an influence on the levitation force, as shown in Fig. 7. From Fig. 7 the following conclusions may be drawn:

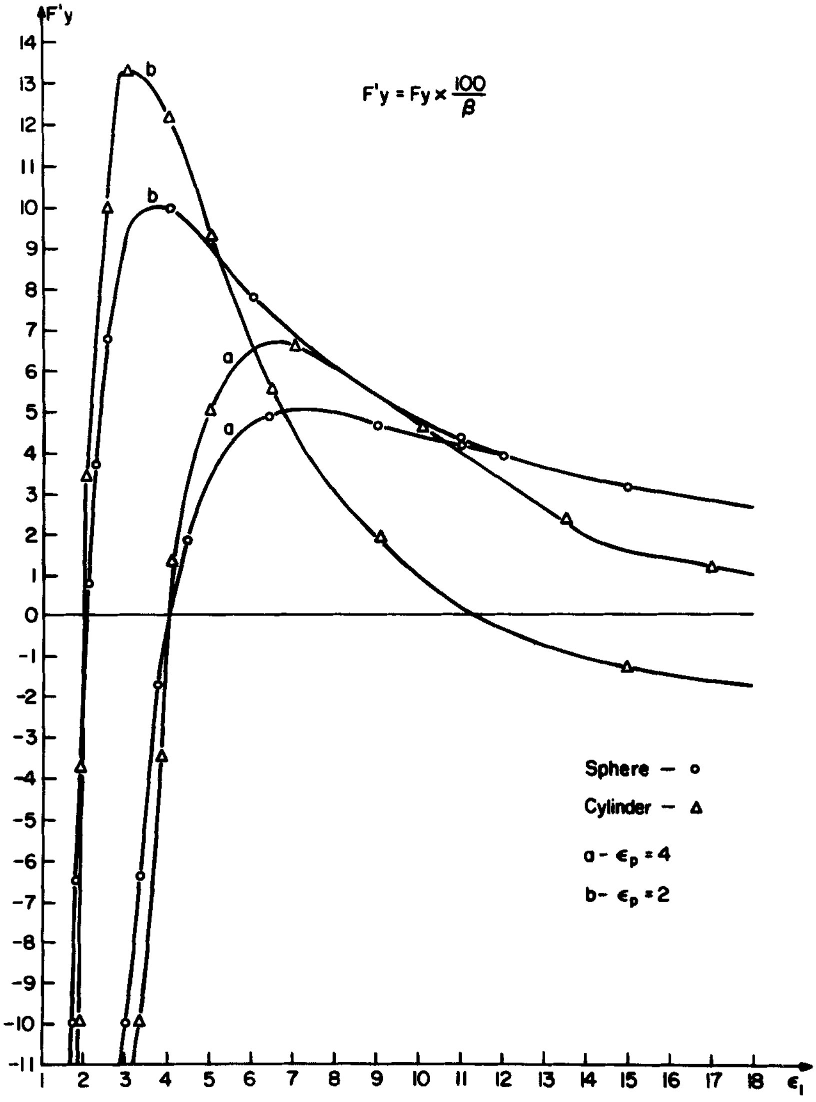

FIG. 7. Levitation force as function of particle shape, dielectric constant of particle, and dielectric constant of liquid.

(a) For a given value of γ_e there is a maximum levitation force, the value of γ_e, in turn, being determined by the dielectric constant of the particle. For example, with a spherical particle of $\varepsilon_p = 4$, a dielectric liquid having $\varepsilon_l = 7.5$ is needed in order to achieve maximum levitation force, which leads to $\gamma_e = 0.184$. If the particle has $\varepsilon_p = 2$, a dielectric liquid having $\varepsilon_l = 4$ is needed for maximum levitation force, and $\gamma_e = 0.2$.

(b) Particles with equal dielectric constants but differing in shape cause different levitation forces: a spherical and a cylindrical particle with dielectric constant $\varepsilon_p = 2$ in a medium of $\varepsilon_l = 3$ cause different levitation forces, the larger one acting on the cylindrical particle. This difference is very considerable—in this case it is higher by 40%.

(c) When a cylindrical particle of $\varepsilon_p = 2$ is immersed in a liquid, the dielectric constant of which is higher than $\varepsilon_l = 11$, the levitation force is negative although $\varepsilon_l > \varepsilon_p$. This can be explained by the fact that, due to the presence of the particle, the converging field is very high, so that the gradient caused by the electrodes is canceled out.

CONCLUSIONS AND DISCUSSION

The purpose of this work was to compare two methods for particle levitation, references to which are often found in the literature.

As a first step it was proved that the magnetohydrostatic system produces a high apparent density which is applicable to particles of a wide range of densities, while the wet dielectric separation system is adversely affected by the physical properties of the dielectric liquid and produces only insignificant apparent densities when conventional electric fields (40 kV/cm) are applied. Another point of view from which the M.H.S. system is preferable is that of the effect of particle size and shape. While these parameters exert almost no influence on the levitation force in the M.H.S. method, they heavily affect it in the W.D.S. method, the changes caused by the parameters mentioned being in the range of 40% of the levitation force.

Furthermore, the levitation force in the W.D.S. system has a maximum value which depends on the dielectric constants of the liquid and of the particle. In some cases (e.g., cylindrical particles) an opposite levitation force is caused, since the electric field is diverging into the liquid due to the presence of the particle (and not converging on the particle), and the diverging field cancels the gradient produced by the curved electrodes.

For that reason and those mentioned before, it becomes obvious that the W.D.S. method is not practical unless high-gradient electric fields are applied. It is therefore recommended that the M.H.S. method be used for the levitation of particles.

APPENDIX A: SOLUTION OF EQ. (11)

Integration of Eq. (11) with respect to ϕ yields

$$\int_0^{2\pi} d\phi = 2\pi \tag{A-1}$$

and with respect to θ yields

$$\int_0^{\pi} (l^2 + x^2 - 2lx\cos\theta)^{k-2}(l - x\cos\theta)\sin\theta\, d\theta$$
$$= \int_0^{\pi} \left[\frac{(l^2 + x^2 - 2xl\cos\theta)^{k-1}}{2x(k-1)} - \frac{\cos\theta(l^2 + x^2 - 2lx\cos\theta)^{k-1}}{2l(k-1)} - \frac{(l^2 + x^2 - 2lx\cos\theta)^4}{4l^2xk(k-1)}\right] \tag{A-2}$$

Whence

$$(F_m)_y = \frac{\pi(\chi_l - \chi_p)A^2K}{2}\int_0^R \left[\frac{2kx}{l}(l + x)^{2k-1} - \frac{x}{l^2}(l + x)^{2k} - \frac{2kx}{l}(l - x)^{2k-1} + \frac{x}{l^2}(l - x)^{2k}\right]dx \tag{A-3}$$

and finally:

$$(F_m)_y = \frac{\pi(\chi_l - \chi_p)A^2K}{2}\left[\frac{(l - R)^{2(k+1)} - (l + R)^{2(k+1)}}{2l^2(k+1)} + \frac{(l + R)^{2k+1} - (l - R)^{2k+1}}{l} + (l - R)^{2k} - (l + R)^{2k}\right] \tag{A-4}$$

APPENDIX B: SOLUTION OF EQ. (22)

The integral in Eq. (22) can be written thus:

$$I = \int_0^{\pi} [(a^2 + l^2)\sin\theta\cos\theta + 2\gamma(a^2 + l^2)\sin\theta\cos\theta - 6\gamma(a^2 + l^2)\sin^3\theta\cos\theta - 2al\sin\theta\cos^2\theta - 4\gamma al\cos^2\theta\sin\theta + 12al\cos^2\theta\sin^3\theta]\, d\theta \tag{B-1}$$

$$I_1 = \int_0^{\pi} (a^2 + l^2) \sin\theta \cos\theta \, d\theta = 0 \quad \text{(B-2)}$$

$$I_2 = \int_0^{\pi} 2\gamma(a^2 + l^2) \sin\theta \cos\theta \, d\theta = 0 \quad \text{(B-3)}$$

$$I_3 = \int_0^{\pi} -6\gamma(a^2 + l^2) \sin^3\theta \cos\theta \, d\theta = 0 \quad \text{(B-4)}$$

$$I_4 = \int_0^{\pi} -2al \sin\theta \cos^2\theta \, d\theta = -(4/3)al \quad \text{(B-5)}$$

$$I_5 = \int_0^{\pi} -4\gamma al \sin\theta \cos^2\theta \, d\theta = -(8/3)\gamma al \quad \text{(B-6)}$$

$$I_6 = \int_0^{\pi} 12al \cos^2\theta \sin^3\theta \, d\theta = (48/15)al \quad \text{(B-7)}$$

Equation (B-1) reduces to the following form:

$$I = (2/3)(2.8 - 4\gamma) \quad \text{(B-8)}$$

SYMBOLS

- E electric field intensity
- f force per unit volume
- F force
- H magnetic field intensity
- k dimensionless number
- p hydrostatic pressure
- V particle volume
- μ magnetic permeability
- ρ density
- χ magnetic susceptibility
- ε dielectric constant

Subscripts

- e electric
- l liquid
- m magnetic
- p particle
- v volume
- y gravity direction
- 0 base value

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